

SINGLE VARIABLE
CALCULUS

EARLY TRANSCENDENTALS

SIXTH EDITION

JAMES STEWART

McMASTER UNIVERSITY

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Single Variable Calculus: Early Transcendentals, Sixth Edition

James Stewart

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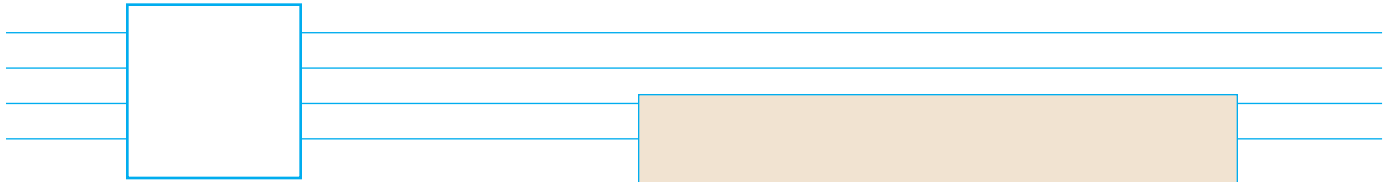
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FOR SALLY AND DON

FOR ALAN AND SHARON

FOR KELLY, KIM, AND CALLUM

FOR JACKIE AND NINO



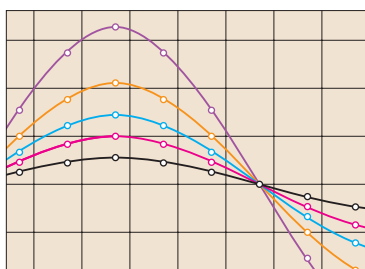
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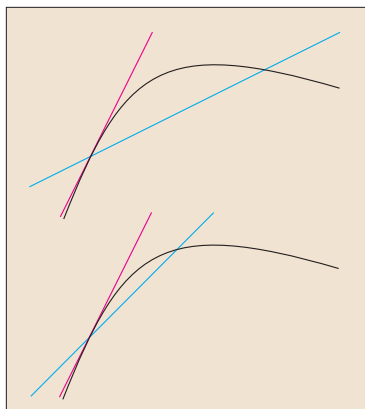


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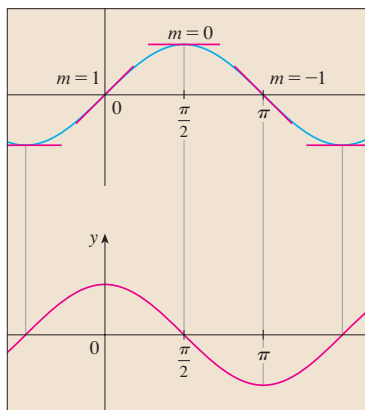
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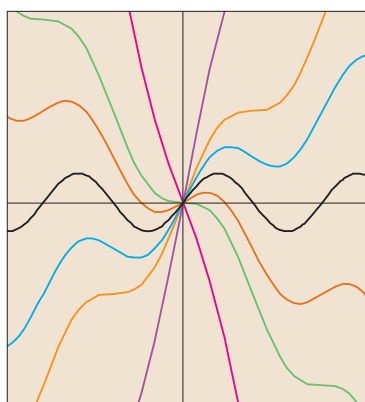
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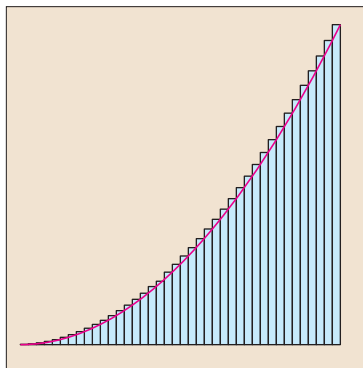
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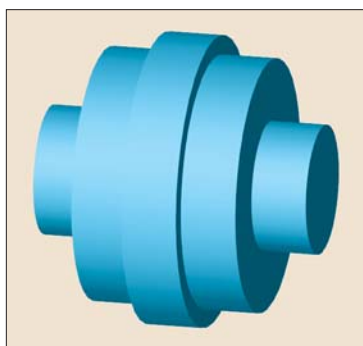
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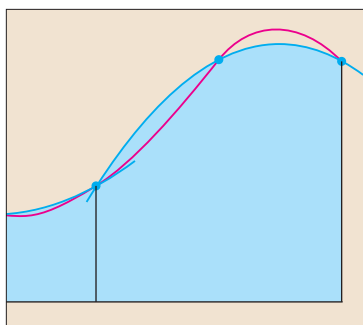
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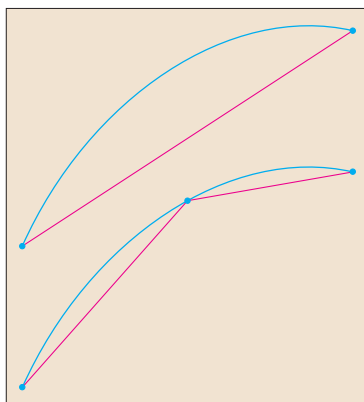
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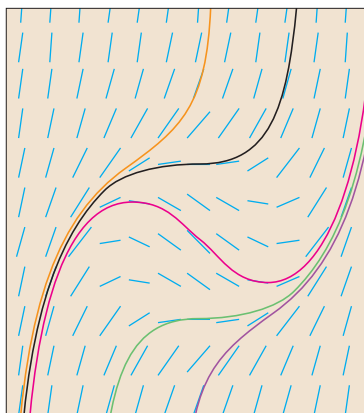


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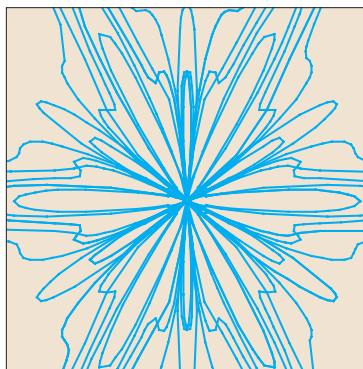
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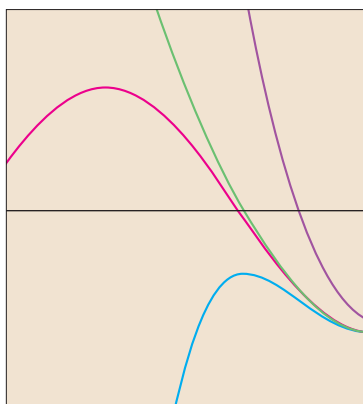
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PREFACE

A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.

GEORGE POLYA

The art of teaching, Mark Van Doren said, is the art of assisting discovery. I have tried to write a book that assists students in discovering calculus—both for its practical power and its surprising beauty. In this edition, as in the first five editions, I aim to convey to the student a sense of the utility of calculus and develop technical competence, but I also strive to give some appreciation for the intrinsic beauty of the subject. Newton undoubtedly experienced a sense of triumph when he made his great discoveries. I want students to share some of that excitement.

The emphasis is on understanding concepts. I think that nearly everybody agrees that this should be the primary goal of calculus instruction. In fact, the impetus for the current calculus reform movement came from the Tulane Conference in 1986, which formulated as their first recommendation:

Focus on conceptual understanding.

I have tried to implement this goal through the *Rule of Three*: “Topics should be presented geometrically, numerically, and algebraically.” Visualization, numerical and graphical experimentation, and other approaches have changed how we teach conceptual reasoning in fundamental ways. More recently, the Rule of Three has been expanded to become the *Rule of Four* by emphasizing the verbal, or descriptive, point of view as well.

In writing the sixth edition my premise has been that it is possible to achieve conceptual understanding and still retain the best traditions of traditional calculus. The book contains elements of reform, but within the context of a traditional curriculum.



ALTERNATIVE VERSIONS

I have written several other calculus textbooks that might be preferable for some instructors. Most of them also come in single variable and multivariable versions.

- *Calculus*, Sixth Edition, is similar to the present textbook except that the exponential, logarithmic, and inverse trigonometric functions are covered in the second semester.
- *Essential Calculus* is a much briefer book (800 pages), though it contains almost all of the topics in the present text. The relative brevity is achieved through briefer exposition of some topics and putting some features on the website.
- *Essential Calculus: Early Transcendentals* resembles *Essential Calculus*, but the exponential, logarithmic, and inverse trigonometric functions are covered in Chapter 3.


- *Calculus: Concepts and Contexts*, Third Edition, emphasizes conceptual understanding even more strongly than this book. The coverage of topics is not encyclopedic and the material on transcendental functions and on parametric equations is woven throughout the book instead of being treated in separate chapters.
- *Calculus: Early Vectors* introduces vectors and vector functions in the first semester and integrates them throughout the book. It is suitable for students taking Engineering and Physics courses concurrently with calculus.



WHAT'S NEW IN THE SIXTH EDITION?

Here are some of the changes for the sixth edition of *Single Variable Calculus: Early Transcendentals*:

- At the beginning of the book there are four diagnostic tests, in Basic Algebra, Analytic Geometry, Functions, and Trigonometry. Answers are given and students who don't do well are referred to where they should seek help (Appendixes, review sections of Chapter 1, and the website).
- In response to requests of several users, the material motivating the derivative is briefer: Sections 2.7 and 2.8 are combined into a single section called Derivatives and Rates of Change.
- The section on Higher Derivatives in Chapter 3 has disappeared and that material is integrated into various sections in Chapters 2 and 3.
- Instructors who do not cover the chapter on differential equations have commented that the section on Exponential Growth and Decay was inconveniently located there. Accordingly, it is moved earlier in the book, to Chapter 3. This move precipitates a reorganization of Chapters 3 and 9.
- Sections 4.7 and 4.8 are merged into a single section, with a briefer treatment of optimization problems in business and economics.
- Sections 11.10 and 11.11 are merged into a single section. I had previously featured the binomial series in its own section to emphasize its importance. But I learned that some instructors were omitting that section, so I have decided to incorporate binomial series into 11.10.
- The material on cylindrical and spherical coordinates (formerly Section 12.7) is moved to Chapter 15, where it is introduced in the context of evaluating triple integrals.
- New phrases and margin notes have been added to clarify the exposition.
- A number of pieces of art have been redrawn.
- The data in examples and exercises have been updated to be more timely.
- Many examples have been added or changed. For instance, Example 2 on page 185 was changed because students are often baffled when they see arbitrary constants in a problem and I wanted to give an example in which they occur.
- Extra steps have been provided in some of the existing examples.
- More than 25% of the exercises in each chapter are new. Here are a few of my favorites: 3.1.79, 3.1.80, 4.3.62, 4.3.83, 4.6.36 and 11.11.30.

- There are also some good new problems in the Problems Plus sections. See, for instance, Problems 2 and 13 on page 413, Problem 13 on page 450, and Problem 24 on page 763.
- The new project on page 550, *Complementary Coffee Cups*, comes from an article by Thomas Banchoff in which he wondered which of two coffee cups, whose convex and concave profiles fit together snugly, would hold more coffee.
- Tools for Enriching Calculus (TEC) has been completely redesigned and is accessible on the Internet at www.stewartcalculus.com. It now includes what we call Visuals, brief animations of various figures in the text. In addition there are now Visuals, Modules, and Homework Hints for the multivariable chapters. See the description on page xiv.
- The symbol  has been placed beside examples (an average of three per section) for which there are videos of instructors explaining the example in more detail. This material is also available on DVD. See the description on page xx.



WHAT'S NEW IN THE SIXTH EDITION?

CONCEPTUAL EXERCISES

The most important way to foster conceptual understanding is through the problems that we assign. To that end I have devised various types of problems. Some exercise sets begin with requests to explain the meanings of the basic concepts of the section. (See, for instance, the first few exercises in Sections 2.2, 2.5, and 11.2.) Similarly, all the review sections begin with a *Concept Check* and a *True-False Quiz*. Other exercises test conceptual understanding through graphs or tables (see Exercises 2.7.17, 2.8.33–38, 2.8.41–44, 9.1.11–12, 10.1.24–27, and 11.10.2).

Another type of exercise uses verbal description to test conceptual understanding (see Exercises 2.5.8, 2.8.56, 4.3.63–64, and 7.8.67). I particularly value problems that combine and compare graphical, numerical, and algebraic approaches (see Exercises 2.6.37–38, 3.7.25, and 9.4.2).

GRADED EXERCISE SETS



Each exercise set is carefully graded, progressing from basic conceptual exercises and skill-development problems to might surprise you.) The project after Section 14.8 uses Lagrange multipliers to determine the masses of the three stages of a rocket so as to minimize the total mass while enabling the rocket to reach a desired velocity. *Laboratory Projects* involve technology; the one following Section 10.2 shows how to use Bézier curves to design shapes that represent letters for a laser printer. *Writing Projects* ask students to compare present-day methods with those of the founders of calculus—Fermat's method for finding tangents, for instance. Suggested references are supplied. *Discovery Projects* anticipate results to be discussed later or encourage discovery through pattern recognition (see the one following Section 7.6). Additional projects can be found in the *Instructor's Guide* (see, for instance, Group Exercise 5.1: Position from Samples).

PROBLEM SOLVING

Students usually have difficulties with problems for which there is no single well-defined procedure for obtaining the answer. I think nobody has improved very much on George Polya's four-stage problem-solving strategy and, accordingly, I have included a version of his problem-solving principles following Chapter 1. They are applied, both explicitly and implicitly, throughout the book. After the other chapters I have placed sections called *Problems Plus*, which feature examples of how to tackle challenging calculus problems. In selecting the varied problems for these sections I kept in mind the following advice from

David Hilbert: “A mathematical problem should be difficult in order to entice us, yet not inaccessible lest it mock our efforts.” When I put these challenging problems on assignments and tests I grade them in a different way. Here I reward a student significantly for ideas toward a solution and for recognizing which problem-solving principles are relevant.

TECHNOLOGY

The availability of technology makes it not less important but more important to clearly understand the concepts that underlie the images on the screen. But, when properly used, graphing calculators and computers are powerful tools for discovering and understanding those concepts. This textbook can be used either with or without technology and I use two special symbols to indicate clearly when a particular type of machine is required. The icon  indicates an exercise that definitely requires the use of such technology, but that is not to say that it can't be used on the other exercises as well. The symbol  is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, Mathematica, or the TI-89/92) are required. But technology doesn't make pencil and paper obsolete. Hand calculation and sketches are often preferable to technology for illustrating and reinforcing some concepts. Both instructors and students need to develop the ability to decide where the hand or the machine is appropriate.

TOOLS FOR ENRICHING™ CALCULUS

TEC is a companion to the text and is intended to enrich and complement its contents. (It is now accessible from the Internet at www.stewartcalculus.com.) Developed by Harvey Keynes, Dan Clegg, Hubert Hohn, and myself, TEC uses a discovery and exploratory approach. In sections of the book where technology is particularly appropriate, marginal icons direct students to TEC modules that provide a laboratory environment in which they can explore the topic in different ways and at different levels. *Visuals* are animations of figures in text; *Modules* are more elaborate activities and include exercises. Instructors can choose to become involved at several different levels, ranging from simply encouraging students to use the Visuals and Modules for independent exploration, to assigning specific exercises from those included with each Module, or to creating additional exercises, labs, and projects that make use of the Visuals and Modules.

TEC also includes *Homework Hints* for representative exercises (usually odd-numbered) in every section of the text, indicated by printing the exercise number in red. These hints are usually presented in the form of questions and try to imitate an effective teaching assistant by functioning as a silent tutor. They are constructed so as not to reveal any more of the actual solution than is minimally necessary to make further progress.

ENHANCED WEBASSIGN

Technology is having an impact on the way homework is assigned to students, particularly in large classes. The use of online homework is growing and its appeal depends on ease of use, grading precision, and reliability. With the sixth edition we have been working with the calculus community and WebAssign to develop an online homework system. Up to 70% of the exercises in each section are assignable as online homework, including free response, multiple choice, and multi-part formats. Some questions are multi-part problems based on simulations of the TEC Modules.

The system also includes Active Examples, in which students are guided in step-by-step tutorials through text examples, with links to the textbook and to video solutions.

WEBSITE: www.stewartcalculus.com

This site has been renovated and now includes the following.

- Algebra Review
- Lies My Calculator and Computer Told Me
- History of Mathematics, with links to the better historical websites

- Additional Topics (complete with exercise sets): Fourier Sets, Formulas for the Remainder Term in Taylor Series, Rotation of Axes
- Archived Problems (Drill exercises that appeared in previous editions, together with their solutions)
- Challenge Problems (some from the Problems Plus sections from prior editions)
- Links, for particular topics, to outside web resources
- The complete Tools for Enriching Calculus (TEC) Modules, Visuals, and Homework Hints



CONTENT

Diagnostic Tests	The book begins with four diagnostic tests, in Basic Algebra, Analytic Geometry, Functions, and Trigonometry.
A Preview of Calculus	This is an overview of the subject and includes a list of questions to motivate the study of calculus.
1 ■ Functions and Models	From the beginning, multiple representations of functions are stressed: verbal, numerical, visual, and algebraic. A discussion of mathematical models leads to a review of the standard functions, including exponential and logarithmic functions, from these four points of view.
2 ■ Limits and Derivatives	The material on limits is motivated by a prior discussion of the tangent and velocity problems. Limits are treated from descriptive, graphical, numerical, and algebraic points of view. Section 2.4, on the precise ϵ - δ definition of a limit, is an optional section. Sections 2.7 and 2.8 deal with derivatives (especially with functions defined graphically and numerically) before the differentiation rules are covered in Chapter 3. Here the examples and exercises explore the meanings of derivatives in various contexts. Higher derivatives are now introduced in Section 2.8.
3 ■ Differentiation Rules	All the basic functions, including exponential, logarithmic, and inverse trigonometric functions, are differentiated here. When derivatives are computed in applied situations, students are asked to explain their meanings. Exponential growth and decay are now covered in this chapter.
4 ■ Applications of Differentiation	The basic facts concerning extreme values and shapes of curves are deduced from the Mean Value Theorem. Graphing with technology emphasizes the interaction between calculus and calculators and the analysis of families of curves. Some substantial optimization problems are provided, including an explanation of why you need to raise your head 42° to see the top of a rainbow.
5 ■ Integrals	The area problem and the distance problem serve to motivate the definite integral, with sigma notation introduced as needed. (Full coverage of sigma notation is provided in Appendix E.) Emphasis is placed on explaining the meanings of integrals in various contexts and on estimating their values from graphs and tables.
6 ■ Applications of Integration	Here I present the applications of integration—area, volume, work, average value—that can reasonably be done without specialized techniques of integration. General methods are emphasized. The goal is for students to be able to divide a quantity into small pieces, estimate with Riemann sums, and recognize the limit as an integral.

- 7 ■ Techniques of Integration** All the standard methods are covered but, of course, the real challenge is to be able to recognize which technique is best used in a given situation. Accordingly, in Section 7.5, I present a strategy for integration. The use of computer algebra systems is discussed in Section 7.6.
- 8 ■ Further Applications of Integration** Here are the applications of integration—arc length and surface area—for which it is useful to have available all the techniques of integration, as well as applications to biology, economics, and physics (hydrostatic force and centers of mass). I have also included a section on probability. There are more applications here than can realistically be covered in a given course. Instructors should select applications suitable for their students and for which they themselves have enthusiasm.
- 9 ■ Differential Equations** Modeling is the theme that unifies this introductory treatment of differential equations. Direction fields and Euler’s method are studied before separable and linear equations are solved explicitly, so that qualitative, numerical, and analytic approaches are given equal consideration. These methods are applied to the exponential, logistic, and other models for population growth. The first four or five sections of this chapter serve as a good introduction to first-order differential equations. An optional final section uses predator-prey models to illustrate systems of differential equations.
- 10 ■ Parametric Equations and Polar Coordinates** This chapter introduces parametric and polar curves and applies the methods of calculus to them. Parametric curves are well suited to laboratory projects; the two presented here involve families of curves and Bézier curves. A brief treatment of conic sections in polar coordinates prepares the way for Kepler’s Laws in Chapter 13.
- 11 ■ Infinite Sequences and Series** The convergence tests have intuitive justifications (see page 697) as well as formal proofs. Numerical estimates of sums of series are based on which test was used to prove convergence. The emphasis is on Taylor series and polynomials and their applications to physics. Error estimates include those from graphing devices.



ANCILLARIES

Calculus, Early Transcendentals, Sixth Edition, is supported by a complete set of ancillaries developed under my direction. Each piece has been designed to enhance student understanding and to facilitate creative instruction. The tables on pages xx–xxi describe each of these ancillaries.



ACKNOWLEDGMENTS

The preparation of this and previous editions has involved much time spent reading the reasoned (but sometimes contradictory) advice from a large number of astute reviewers. I greatly appreciate the time they spent to understand my motivation for the approach taken. I have learned something from each of them.

SIXTH EDITION REVIEWERS

Marilyn Belkin, *Villanova University*
 Philip L. Bowers, *Florida State University*
 Amy Elizabeth Bowman, *University of Alabama in Huntsville*
 M. Hilary Davies, *University of Alaska Anchorage*

Frederick Gass, *Miami University*
 Nets Katz, *Indiana University Bloomington*
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B. D. Aggarwala, *University of Calgary*
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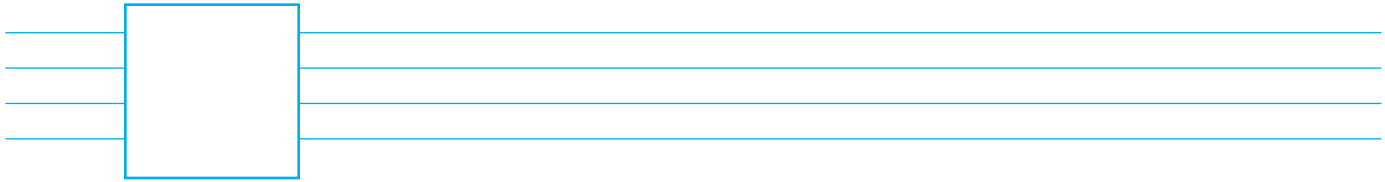
Written to improve algebra and problem-solving skills of students taking a calculus course, every chapter in this companion is keyed to a calculus topic, providing conceptual background and specific algebra techniques needed to understand and solve calculus problems related to that topic. It is designed for calculus courses that integrate the review of precalculus concepts or for individual use.

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by Konrad J. Heuvers, William P. Francis, John H. Kuisti, Deborah F. Lockhart, Daniel S. Moak, and Gene M. Ortner

ISBN 0-534-25248-6

This comprehensive book, designed to supplement the calculus course, provides an introduction to and review of the basic ideas of linear algebra.





TO THE STUDENT


Reading a calculus textbook is different from reading a newspaper or a novel, or even a physics book. Don't be discouraged if you have to read a passage more than once in order to understand it. You should have pencil and paper and calculator at hand to sketch a diagram or make a calculation.

Some students start by trying their homework problems and read the text only if they get stuck on an exercise. I suggest that a far better plan is to read and understand a section of the text before attempting the exercises. In particular, you should look at the definitions to see the exact meanings of the terms. And before you read each example, I suggest that you cover up the solution and try solving the problem yourself. You'll get a lot more from looking at the solution if you do so.

Part of the aim of this course is to train you to think logically. Learn to write the solutions of the exercises in a connected, step-by-step fashion with explanatory sentences—not just a string of disconnected equations or formulas.

The answers to the odd-numbered exercises appear at the back of the book, in Appendix I. Some exercises ask for a verbal explanation or interpretation or description. In such cases there is no single correct way of expressing the answer, so don't worry that you haven't found the definitive answer. In addition, there are often several different forms in which to express a numerical or algebraic answer, so if your answer differs from mine, don't immediately assume you're wrong. For example, if the answer given in the back of the book is $\sqrt{2} - 1$ and you obtain $1/(1 + \sqrt{2})$, then you're right and rationalizing the denominator will show that the answers are equivalent.

The icon  indicates an exercise that definitely requires the use of either a graphing calculator or a computer with graphing software. (Section 1.4 discusses the use of these graphing devices and some of the pitfalls that you may encounter.) But that doesn't mean that graphing devices can't be used to check your work on the other exercises as well. The symbol  is reserved for problems in which the full resources of a computer algebra

system (like Derive, Maple, Mathematica, or the TI-89/92) are required. You will also encounter the symbol , which warns you against committing an error. I have placed this symbol in the margin in situations where I have observed that a large proportion of my students tend to make the same mistake.

Tools for Enriching Calculus, which is a companion to this text and can be accessed from www.stewartcalculus.com, is referred to by means of the symbol **TEC**. It directs you to modules in which you can explore aspects of calculus for which the computer is particularly useful. TEC also provides *Homework Hints* for representative exercises that are indicated by printing the exercise number in red: **15**. These homework hints ask you questions that allow you to make progress toward a solution without actually giving you the answer. You need to pursue each hint in an active manner with pencil and paper to work out the details. If a particular hint doesn't enable you to solve the problem, you can click to reveal the next hint.

An optional CD-ROM that your instructor may have asked you to purchase is the *Interactive Video Skillbuilder*, which contains videos of instructors explaining two or three of the examples in every section of the text. Also on the CD is a video in which I offer advice on how to succeed in your calculus course.

I recommend that you keep this book for reference purposes after you finish the course. Because you will likely forget some of the specific details of calculus, the book will serve as a useful reminder when you need to use calculus in subsequent courses. And, because this book contains more material than can be covered in any one course, it can also serve as a valuable resource for a working scientist or engineer.

Calculus is an exciting subject, justly considered to be one of the greatest achievements of the human intellect. I hope you will discover that it is not only useful but also intrinsically beautiful.

JAMES STEWART